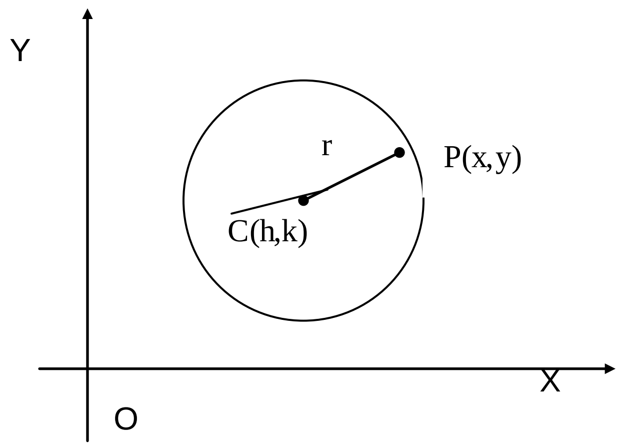
Circles

A circle is defined as the locus of the point, which moves in such a way, that its distance from a fixed point is always constant. The fixed point is called **centre** of the circle and the constant distance is called the **radius** of the circle.

The equation of the circle when the centre and radius are given

Let C (h,k) be the centre and r be the radius of the circle. Let P(x,y) be any point on the circle.

$CP = r \Rightarrow CP^2 = r^2 \Rightarrow (x-h)^2 + (y-k)^2 = r^2$ is the required equation of the circle.



Note :

If the center of the circle is at the origin i.e., C(h,k)=(0,0) then the equation of the circle is $x^2 + y^2 = r^2$

The general equation of the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

Consider the equation $x^2 + y^2 + 2gx + 2fy + c = 0$. This can be written as

$x^2 + y^2 + 2gx + 2fy + g^2 + f^2 = g^2 + f^2 - c$

(i.e) $x^2 + 2gx + g^2 + y^2 + 2fy + f^2 = g^2 + f^2 - c$

$$(x + g)^2 + (y + f)^2 = \left(\sqrt{g^2 + f^2 - c}\right)^2$$

$$[x-(-g)]^2 + [y-(-f)]^2 = \left(\sqrt{g^2 + f^2 - c}\right)^2$$

This is of the form $(x-h)^2 + (y-k)^2 = r^2$

$\therefore$ The considered equation represents a circle with centre (-g,-f) and radius $\sqrt{g^2 + f^2 - c}$

$\therefore$ The general equation of the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

where

c = The Center of the circle whose coordinates are (-g,-f)

r = The radius of the circle = $\sqrt{g^2 + f^2 - c}$

Note

The general second degree equation

$$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$$

Represents a circle if

(i) $a = b$ i.e., coefficient of x^2 = coefficient of y^2

(ii) $h = 0$ i.e., no xy term