Lecture.20

2^3 factorial experiments in RBD – lay out – analysis

2^3 Factorial Experiment in RBD

2^3 factorial experiment mean three factors each at two levels. Suppose the three factors are A, B and C are tried with two levels the total number of combinations will be eight i.e. a0b0c0, a0b0c1, a0b1c0, a0b1c1, a1b0c0, a1b0c1, a1b1c0 and a1b1c1.

The allotment of these eight treatment combinations will be as allotted in RBD. That is each block is divided into eight experimental units. By using the random numbers these eight combinations are allotted at random for each block separately.

The analysis of variance table for three factors A with a levels, B with b levels and C with c levels with r replications tried in RBD will be as follows:

Sources of Variation	**d.f.**	**SS**	**MS**	**F**
Replications	r-1	RSS	RMS	
Factor A	a-1	ASS	AMS	AMS / EMS
Factor B	b-1	BSS	BMS	BMS / EMS
Factor C	c-1	CSS	CMS	CMS / EMS
AB	(a-1)(b-1)	ABSS	ABMS	ABMS / EMS
AC	(a-1)(c-1)	ACSS	ACMS	ACMS / EMS
BC	(b-1)(c-1)	BCSS	BCMS	BCMS / EMS
ABC	(a-1)(b-1)(c-1)	ABCSS	ABCMS	ABCMS / EMS
Error	(r-1)(abc-1)	ESS	EMS	
Total	rabc-1	TSS		

Analysis

1. Arrange the results as per treatment combinations and replications.

Treatment combination	Replication R1	R2	R3	…	Treatment Total
a0b0c0					T1
a0b0c1					T2
a0b1c0					T3
a0b1c1					T4
a1b0c0					T5
a1b0c1					T6
a1b1c0					T7
a1b1c1					T8

As in the previous designs calculate the replication totals to calculate the CF, RSS, TSS, overall TrSS in the usual way. To calculate ASS, BSS, CSS, ABSS, ACSS, BCSS and ABCSS, form three two way tables A X B, AXC and BXC.

AXB two way table can be formed by taking the levels of A in rows and levels of B in the columns. To get the values in this table the missing factor is replication. That is by adding over replication we can form this table.

A X B Two way table

B / A	b_0	b_1	Total
a_0	$a_0 b_0$	$a_0 b_1$	A_0
a_1	$a_1 b_0$	$a_1 b_1$	A_1

Total	B_0	B_1	Grand Total

$$\text{ASS}=\frac{A_0^2 + A_1^2}{b \times c \times r} - CF$$

$$\text{BSS} = \frac{B_0^2 + B_1^2}{a \times c \times r} - CF$$

$$\text{ABSS} = \frac{(a_0 b_0)^2 + (a_0 b_1)^2 + (a_1 b_0)^2 + (a_1 b_1)^2}{c \times r} - CF - ASS - BSS$$

A X C two way table can be formed by taking the levels of A in rows and levels of C in the columns

A X C Two way table

C / A	c_0	c_1	Total
a_0	$a_0 c_0$	$a_0 c_1$	A_0
a_1	$a_1 c_0$	$a_1 c_1$	A_1
Total	C_0	C_1	Grand Total

$$\text{CSS} = \frac{C_0^2 + C_1^2}{a \times b \times r} - CF$$

$$\text{ACSS} = \frac{(a_0 c_0)^2 + (a_0 c_1)^2 + (a_1 c_0)^2 + (a_1 c_1)^2}{b \times r} - CF - ASS - CSS$$

B X C two way table can be formed by taking the levels of B in rows and levels of C in the columns

B X C Two way table

C / B	c_0	c_1	Total

b_0	$b_0 c_0$	$b_0 c_1$	B_0
b_1	$b_1 c_0$	$b_1 c_1$	B_1
Total	C_0	C_1	Grand Total

$$\text{BCSS} = \frac{(b_0c_0)^2 + (b_0c_1)^2 + (b_1c_0)^2 + (b_1c_1)^2}{a \times r} - CF - BSS - CSS$$

$$\text{ABCSS} = \frac{(a_0b_0c_0)^2 + (a_0b_0c_1)^2 + (a_0b_1c_0)^2 + (a_0b_1c_1)^2 + (a_1b_0c_0)^2 + (a_1b_0c_1)^2 + (a_1b_1c_0)^2 + (a_1b_1c_1)^2}{r}$$

-CF-ASS-BSS-CSS-ABSS-ACSS-BCSS

ESS = TSS-RSS- ASS-BSS-CSS-ABSS-ACSS-BCSS-ABCSS

By substituting the above values in the ANOVA table corresponding to the columns sum of squares, the mean squares and F value can be calculated.

Questions

1. 2^3 factorial experiment means two factors each at

a) two levels b**)** three levels c) four levels d) one level

Ans: three levels

2. If the total number of combinations are eight then each block is divided into _____ experimental units

a) Two b) three c) four d) eight

Ans: eight

3. If the three factors are A, B and C are tried with three levels the total number of combinations will be twenty seven.

Ans: True

4. The error degrees of freedom for an experiment with one factor at 2 levels, second factor at 3 levels and the third at 3 levels and 3 replications will be 34.

Ans: True

5. In a three factor experiment the highest order interaction will be the two factor interaction.

Ans: False

6. In a factorial experiment with three factors the treatment sum of squares will be split up into factor 1, factor 2 and factor 1 x factor 2 interaction sum of squares.

Ans: False

7. How to calculate the three factor interaction sum of squares in a three factor experiment.

8. In a factorial experiment what is the total number of experimental units when there are 3 replications, 4 levels for factor A and 3 levels for factor B and C.

9. Furnish the ANOVA table for a three factor experiment with factor A at a levels, Factor B at b levels and factor C at c levels and the number of replication r.

10. Explain the procedure of forming a B X C two way table and calculating the factor BSS, CSS and B X CSS.