Lesson43

Non-Homogeneous Linear Equation

43.1 Complementary Function and Particular Solution

Consider the non-homogeneous linear equation

$$f(D,D')z = F(x,y) \text{ where } f(D,D') = \prod_{r=1}^{n} D_r - mD'_r - C_r \, ;$$

for some fixed r, the solution may be written as

$$\frac{dx}{1} = \frac{dy}{-m} = \frac{dz}{cz} \Rightarrow y + mx = a \, , \; z = be^{cx} \, .$$

Example1: $\left(D^2 + 2DD' + D'^2 - 2D - 2D'\right)z = \sin\left(x + 2y\right)$

$\left(D + D'\right)\left(D + D' - 2\right)z = \sin\left(x + 2y\right)$

Solution corresponding to the factor $\left(D + D' - 2\right)$ is:

$$z = e^{2x}\phi\left(y - x\right)$$

and the complementary function is: $\phi_1\left(y - x\right) + e^{2x}\phi\left(y - x\right)$.

The Particular Integral is

$$\frac{1}{\left(D^2 + 2DD' + D'^2 - 2D - 2D'\right)}\sin\left(x + 2y\right)$$

$$= -\frac{1}{2(D+D')+9}\sin(x+2y)$$

$$= \frac{-2(D+D')-9}{4\left(D^2+2DD'+D'^2\right)-81}\sin(x+2y)$$

$$= \frac{1}{39}\left[2\cos(x+2y)-3\sin(x+2y)\right]$$

Exercises: Solve the following non-homogeneous equations

1. $\left(D^2+DD'+D'-1\right)z = e^{-x}$
2. $(D+D'-1)(D+2D'-3)z = 4+3x+6y$
3. $\left(D'+DD'+D'\right)z = x^2+y^2$
4. $\left(2DD'+D'^2-3D'\right)z = 3\cos(3x-2y)$

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References

Ian Sneddon, (1957). Elements of Partial Differential Equations. McGraw-Hill, Singapore

Amaranath.T, (2003). An Elementary Course in Partial Differential Equations.Narosa Publishing House, New Delhi

Suggested Reading

I.P. Stavroulakis, Stephen a tersian, (2003). Partial Differential Equations. Allied Publishers Pvt. Limited, New Delhi

J. David Logan, (2004). Partial Differential Equations. Springer(India) Private Ltd. New Delhi