Lesson 5

Secant and Regula-Falsi Methods

5.1 Introduction

A root of the algebraic equation $f(x)=0$ can be obtained by using the iteration methods based on the first degree equation. We approximate the given $f(x)$ by a first degree equation in the neighborhood of the root of $f(x)=0$.

We may write the first degree equation as:

$$f(x)=a_1x+a_0 \tag{5.1}$$

with $a_1 \neq 0$ and a_0 and a_1 are arbitrary parameters to be determined by posing two conditions on $f(x)$ or its derivative at two different x locations on the curve.

5.2 Scant Method

Take two approximations x_{k-1} and x_k to the root ξ of $f(x)=0$. We determine the constants a_0 and a_1 in equation (5.1) by using the conditions

$$f_{k-1}=a_1x_{k-1}+a_0 \tag{5.2}$$

and

$$f_k=a_1x_k+a_0 \tag{5.3}$$

Where, $f(x_{k-1}) = f_{k-1}$ and $f(x_k) = f_k$.

Solving equations (5.2) and (5.3), we get,

$$a_0 = \frac{\left(x_k f_{k-1} - x_{k-1} f_k\right)}{\left(x_k - x_{k-1}\right)} \tag{4}$$

and

$$a_1 = \frac{\left(f_k - f_{k-1}\right)}{\left(x_k - x_{k-1}\right)} \tag{5}$$

The solution of (5.1) is

$$x = -\frac{a_0}{a_1} \tag{6}$$

Now the new approximation x_{k+1} based on two initial approximations x_{k-1} and x_k is written as equation (5.6) as:

$$x_{k+1} = \frac{\left(x_k f_{k-1} - x_{k-1} f_k\right)}{\left(f_k - f_{k-1}\right)}$$

This may be rewritten as:

$$x_{k+1} = x_k - \frac{\left(x_k - x_{k-1}\right)}{\left(f_k - f_{k-1}\right)} f_k,\ k = 1, 2, \ldots \tag{7}$$

Here iteration function $\varphi(x_k, x_{k-1}) = x_k - \frac{(x_k - x_{k-1})}{(f_k - f_{k-1})} f_k$ generates a sequence of iterates $\{x_{k+1}\}_{k=1}^{\infty}$.

5.2.1 Algorithm

1. Start with (x_0, f_0) and (x_1, f_1).
2. Generate x_2 using (7) and compute $f_2 = f(x_2)$.
3. Use (x_1, f_1) and (x_2, f_2) to compute x_3.
4. Repeat steps (2) & (3) until the difference between two successive approximations x_{k+1} and x_k is less than the error tolerance.

The method fails if at any stage of computation $f(x_k) = f(x_{k-1})$ (see equation 5.7).

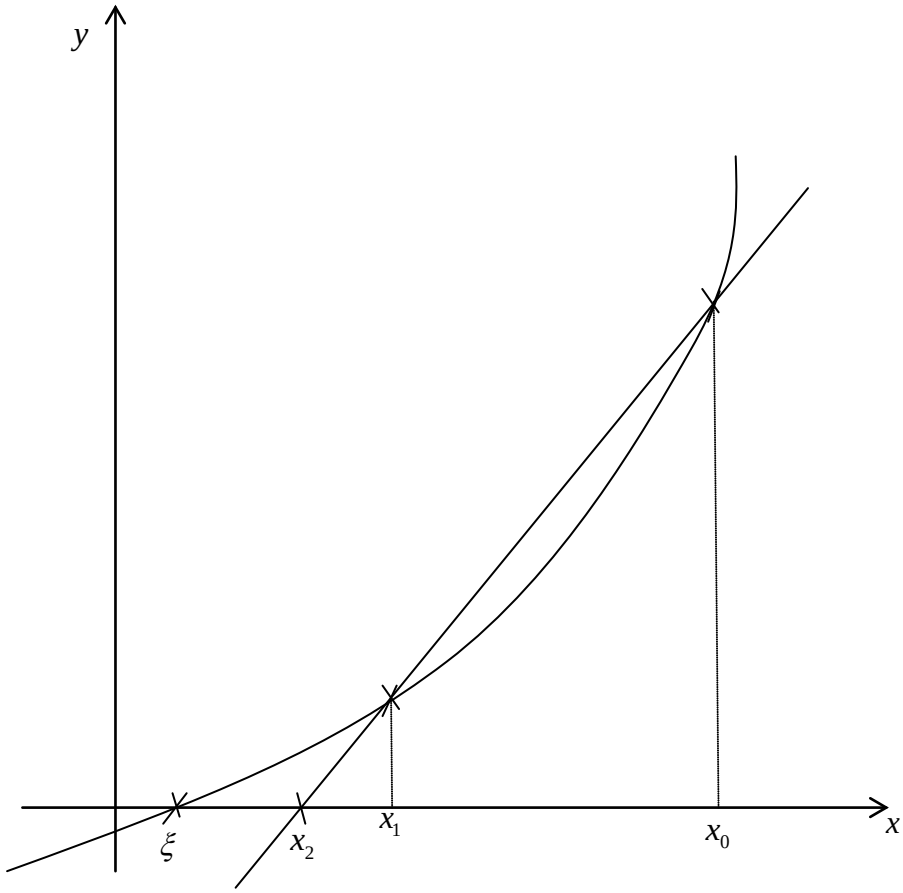

Fig. 5.1. Secant Method – a Graphical Representation.

5.3 Regula-Falsi Method

The initial approximations x_{k-1} and x_k taken in the secant method are arbitrary. If we impose a condition on these initial approximations such that $f(x_k).f(x_{k-1})<0$ the method given by equation (5.7) satisfying this condition is called the Regula-falsi method. Then the root of $f(x)=0$ lies in between these two values x_k and x_{k-1}. Now draw a chord joining the points (x_{k-1}, f_{k-1}) and (x_k, f_k). This chord intersects the $x-$axis, say at $x=x_{k+1}$. Now look for $f(x_{k+1}).f(x_k)<0$ or $f(x_{k+1}).f(x_{k-1})<0$. One of these two will hold. Without loss of generality, say $f(x_{k+1}).f(x_k)<0$, then

join the points (x_k, f_k) and (x_{k+1}, f_{k+1}) by a chord, this chord intersects $x - axis$ say at $x = x_{k+2}$. The above procedure is repeated till we reach the root of $f(x) = 0$. In this procedure we are sure of convergence of the sequence of iterates generated using equation (5.7).

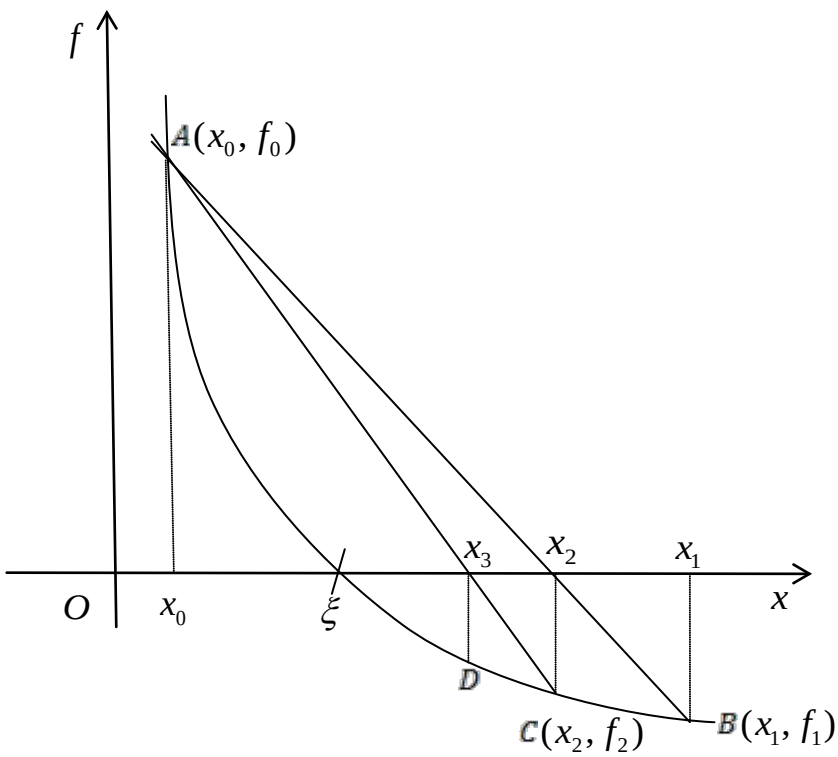

Fig. 5.2. Regula - Falsi Method – a Graphical Representation.

Example 1: Find the real root of $x^3 - 2x - 5 = 0$ using the Regula-falsi method.

Solution:

Given $f(x) = x^3 - 2x - 5$. We compute that $f(2) = -1 < 0$ and $f(3) = 16 > 0$.

Take $x_0 = 2, f_0 = -1, x_1 = 3, f_1 = 16$.

Using the method given by (7),

$$x_2 = 3 - \frac{3-2}{16+1}16 = 2.0813, \quad f(x_2) = -0.1472 < 0.$$

The root lies in the interval $(2.083, 3)$, join the points $(2.0813, -0.1472)$ and $(3, 16)$ by the chord, which cuts the $x-$axis at $x_3 = 2.08964$.

Proceeding in this way, we obtain

$x_4 = 2.09274$, $x_5 = 2.09388$, $x_6 = 2.0943$ etc.

The approximate value for the root is taken as $x = 2.0943$.

Now we illustrate this method to find a root of the transcendental equation.

Example 2: Find the root of $\cos x - xe^x = 0$ using the Regula-falsi method correct to four decimal places.

Solution:

Let $f(x) = \cos x - xe^x$, $f(0) = 1$, $f(1) = -2.17798$.

Clearly $f(0) \,.\, f(1) < 0$.

We obtain $x_2 = x_0 - \dfrac{x_1 - x_0}{f_1 - f_0} f_0 \quad = 0.31467$ and $f(0.31467) = 0.51987 > 0$.

The root lies between 0.31467 and 1. Repeating the procedure we obtain the approximations to the roots as

$x_3 = 0.44673$,

$x_4 = 0.49402$,

$x_5 = 0.50995$

$x_6 = 0.51520$, $x_7 = 0.51692$

$x_8 = 0.51748, x_9 = 0.51767$,

$x_{10} = 0.51775, \ldots$

The root correct to 4 decimal places as $\xi = 0.5177$.

Exercises

1. Find a real root of the following equations using the Regula-falsi method:

a) $x \log_0 x = 1.2$

b) $x^4 - 32 = 0$

c) $x^3 - 2x - 5 = 0$

d) $sinx = \dfrac{1}{x}$

2. Solve the above problems using the Secant method.

Keywords: Scant Method, Regula-Falsi Method

References

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Suggested Reading

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