Lesson 15

Everett's Central Difference Interpolation

15.1 Introduction

We have the Gauss forward interpolation formula as

$$y_p = y_0 + p\Delta y_0 + \frac{p(p+1)}{2!}\Delta^2 y_{-1} + \frac{(p+1)p(p-1)}{3!}\Delta^3 y_{-2} + \frac{(p+1)p(p-1)(p-2)}{4!}\Delta^4 y_{-2}$$

$$+\frac{(p+2)(p+1)p(p-1)(p-2)}{5!}\Delta^5 y_{-2} + \ldots \qquad (15.1)$$

15.2 Everett's Formula

Eliminating odd differences $\Delta y_0, \Delta^3 y_{-1}, \Delta^5 y_{-2}$ etc. by

$\Delta y_0 = y_1 - y_0, \Delta^3 y_{-1} = \Delta^2 y_0 - \Delta^2 y_{-1},\ \Delta^5 y_{-2} = \Delta^4 y_{-1} - \Delta^4 y_{-2}$ etc., then (1) becomes

$$y_p = y_0 + p\left(y_1 - y_0\right) + \frac{p(p-1)}{2!}\Delta^2 y_{-1} + \frac{(p+1)p(p-1)}{3!}\left(\Delta^2 y_0 - \Delta^2 y_{-1}\right) +$$

$$\frac{(p+1)p(p-1)(p-2)}{4!}\Delta^4 y_{-2} + \frac{(p+2)(p+1)p(p-1)(p-2)}{5!}\left(\Delta^4 y_{-1} - \Delta^4 y_{-2}\right) + \ldots$$

$$= (1-p)y_0 + py_1 - \frac{p(p-1)(p-2)}{3!}\Delta^2 y_{-1} + \frac{(p+1)p(p-1)}{3!}\Delta^2 y_0 -$$

$$\frac{(p+1)p(p-1)(p-2)(p-3)}{5!}\Delta^4 y_{-2} + \frac{(p+2)(p+1)p(p-1)(p-2)}{5!}\Delta^4 y_{-1} + \ldots$$

$$(15.2)$$

This is known as Everett's formula.

This formula is extensively used as it involves only even differences in and below the central line.

Example 1: Below given data represents the function $f(x) = logx$. Use Everett's formula to find $f(337.5)$:

x:	310	320	330	340	350	360
$f(x)$:	2.49136	2.50515	2.51851	2.53148	2.54407	2.55630

Take the data as:

$x_{-2} = 310,\ f_{-2} = 2.49136,$

$x_{-1} = 320,\ f_{-1} = 2.50515,$

$x_0 = 330,\ f_0 = 2.51851,$

$x_1 = 340,\ f_1 = 2.53148,$

$x_2 = 350,\ f_2 = 2.54407,$

$x_3 = 360,\ f_3 = 2.5630,$

$h = 10,\ p = \dfrac{x - 330}{10}.$

y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
2.49136					
	0.01379				
2.50515		-0.00043			
	0.01336		0.00004		
2.51881		-0.00039		-0.00003	

	0.01297		0.00001		0.00004
2.53148		-0.00038		0.00001	
	0.01259		0.00002		
2.54407		-0.00036			
	0.01223				
2.55630					

Take $x = 337.5$, $p = \dfrac{337.5 - 330}{10} = 0.75$.

To change the terms with negative sign, putting $\mathrm{p} = 1 - \mathrm{q}$ in equation (1), we get

$$y_p = qy_0 + \frac{q(q^2 - 1^2)}{3!}\Delta^2 y_{-1} + \frac{q(q^2 - 1^2)(q^2 - 2^2)}{5!}\Delta^4 y_{-2} + \ldots$$

$$+ py_1 + \frac{p(p^2 - 1^2)}{3!}\Delta^2 y_0 + \frac{p(p^2 - 1^2)(p^2 - 2^2)}{5!}\Delta^4 y_{-1} + \ldots$$

$q = 1 - p = 0.25$.

$\therefore y_p = 0.62963 + 0.00002 - 0.0000002 + 1.89861 + 0.00002 + 0.00000001 = 2.52828$

Exercise:

1. Find $f(25)$ from the data

x:	20	24	28	32
f:	854	3162	3544	3992

using Everett's formula.

Keywords: Everett's Formula, Gauss Forward Interpolation

References

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Suggested Reading

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