Engineering Mathematics III
Module 2: Laplace Transform

QUIZ

1. Laplace transform of the rectangular wave function $W(t-a,b)=\begin{cases}0, & 0\le t, t>b\\ 1, & a<t\le b\end{cases}$ is

(a) $\frac{1}{s}\left(e^{as}-e^{bs}\right)$ (b) $\frac{1}{s}\left(e^{-as}-e^{-bs}\right)$

(c) $\frac{1}{s}\left(e^{-bs}-e^{-as}\right)$ (d) $\frac{1}{s}\left(e^{bs}-e^{as}\right)$

2. Laplace transform of the function $\sin^2 pt$ is

(a) $\frac{4p^2}{s\left(s^2+4p^2\right)}$ (b) $\frac{2p^2}{s\left(s^2+2p^2\right)}$

(c) $\frac{2p^2}{s\left(s^2+4p^2\right)}$ (d) $\frac{2p}{s\left(s^2+4p^2\right)}$

3. Laplace transform of the function $\sin at\cos bt$ is

(a) $\frac{a^2\left(s^2+a^2-b^2\right)}{\left(s^2+(a+b)^2\right)\left(s^2+(a-b)^2\right)}$ (b) $\frac{a\left(s^2+a^2-b^2\right)}{\left(s^2+(a+b)^2\right)\left(s^2+(a-b)^2\right)}$

(c) $\frac{a\left(s^2+a^2-b^2\right)}{\left(s^2+(a+b)\right)\left(s^2+(a-b)\right)}$ (d) $\frac{a\left(s^2+a^2-b^2\right)^2}{\left(s^2+(a+b)^2\right)\left(s^2+(a-b)^2\right)}$

4. Which of the following function is not piecewise continuous?

(a) $f(t)=\frac{1}{t-2},\ t\neq 2$ (b) $f(t)=\begin{cases}2t, & t\le 1\\ 1+t^2, & t>1\end{cases}$

(c) $f(t)=\begin{cases}\frac{1-e^{-t}}{t}, & t\neq 0\\ 0, & t=0\end{cases}$ (d) $f(t)=\begin{cases}t\sin\left(\frac{1}{t}\right), & t\neq 0\\ 0, & t=0\end{cases}$

5. Laplace transform of $t^2 \cos at$ is

(a) $2s\left(s^2+a^2\right)^{-2}-8a^2s\left(s^2+a^2\right)^{-3}$

(b) $2s\left(s^2+a^2\right)^{-2}-8a^2s\left(s^2+a^2\right)^{-2}$

(c) $2s\left(s^2+a^2\right)^{-2}-8a\ s\left(s^2+a^2\right)^{-3}$

(d) $2s^2\left(s^2+a^2\right)^{-2}-8a^2s\left(s^2+a^2\right)^{-3}$

6. Laplace transform of $f(t)=\left|\sin(t)\right|,\ \ t>0$ is

(a) $\dfrac{1-e^{-2\pi s}}{\left(1+e^{-2\pi s}\right)\left(s^2+1\right)}$

(b) $\dfrac{1-e^{-\pi s}}{\left(1+e^{-2\pi s}\right)\left(s^2+1\right)}$

(c) $\dfrac{1+e^{-\pi s}}{\left(1-e^{-\pi s}\right)\left(s^2+1\right)}$

(d) $\dfrac{1-e^{-\pi s}}{\left(1+e^{-\pi s}\right)\left(s^2+1\right)}$

7. Laplace transform of $f(t)=t\,H(t-a),\ \ t>0$ is

(a) $\dfrac{e^{as}}{s^2}\left[1-as\right]$

(b) $\dfrac{e^{as}}{s^2}\left[1+as\right]$

(c) $\dfrac{e^{-as}}{s^2}\left[1-as\right]$

(d) $\dfrac{e^{-as}}{s^2}\left[1+as\right]$

8. Which of the following functions does not possess the Laplace transform?

(a) $e^t \operatorname{erfc}\left(\sqrt{t}\right)$

(b) $\sin\left(e^{t^2}\right)$

(c) e^{t^2}

(d) $te^{t^2}\sin\left(e^{t^2}\right)$

9. The value of $L^{-1}\left[\dfrac{2s+3}{s^2+4}\right]$ is

(a) $\cos 2t+\dfrac{3}{2}\sin 2t$

(b) $2\cos 2t+\dfrac{3}{2}\sin t$

(c) $2\cos 2t+\dfrac{3}{4}\sin 2t$

(d) $2\cos 2t+\dfrac{3}{2}\sin 2t$

10. The inverse Laplace transform of $\dfrac{2-5s}{(s-6)(s^2+11)}$ is

(a) $\dfrac{1}{47}\left[-28e^{6t}+28\cos\sqrt{11}\,t-\dfrac{67}{\sqrt{11}}\sin\sqrt{11}\,t\right]$

(b) $\dfrac{1}{47}\left[-28e^{6t}-28\cos\sqrt{11}\,t+\dfrac{67}{\sqrt{11}}\sin\sqrt{11}\,t\right]$

(c) $\dfrac{1}{47}\left[28e^{6t}+28\cos\sqrt{11}\,t-\dfrac{67}{\sqrt{11}}\sin\sqrt{11}\,t\right]$

(d) $\dfrac{1}{47}\left[28e^{6t}-28\cos\sqrt{11}\,t-\dfrac{67}{\sqrt{11}}\sin\sqrt{11}\,t\right]$

11. The value of $L^{-1}\left[\dfrac{3s-5}{4s^2-4s+1}\right]$ is

(a) $\dfrac{1}{8}e^{\frac{t}{2}}(6-7t)$,

(b) $\dfrac{1}{8}e^{-\frac{t}{2}}(6-7t)$,

(c) $\dfrac{1}{8}e^{\frac{t}{2}}(6+7t)$,

(d) $\dfrac{1}{8}e^{-\frac{t}{2}}(6+7t)$,

12. The value of $F(s)=\dfrac{e^{-\pi s}}{s^2-2}$ is

(a) $\dfrac{1}{\sqrt{2}}H(t-\pi)\cosh\left\{\sqrt{2}(t-\pi)\right\}$

(b) $\dfrac{1}{\sqrt{2}}H(t-\pi)\sin\left\{\sqrt{2}(t-\pi)\right\}$

(c) $\dfrac{1}{\sqrt{2}}H(t-\pi)\cos\left\{\sqrt{2}(t-\pi)\right\}$

(d) $\dfrac{1}{\sqrt{2}}H(t-\pi)\sinh\left\{\sqrt{2}(t-\pi)\right\}$

13. The solution of the initial value problem $\frac{d^2y}{dt^2}+\frac{dy}{dt}=\left(1-H(t-1)\right);\ \ y(0)=1, y'(0)=-1$ is

(a) $y=t-1+2e^{-t}-H\left(t-1\right)\left(t-2+e^{-t+1}\right)$ (b) $y=t-1+2e^{-t}-H\left(t-1\right)\left(t+2+e^{-t+1}\right)$

(c) $y=t-1+2e^{-t}-H\left(t-1\right)\left(t-2+e^{t-1}\right)$ (d) $y=t-1+2e^{-t}-H\left(t-1\right)\left(t+2+e^{t-1}\right)$

14. The solution of the initial value problem

$$\frac{d^2y}{dt^2}+y=f(t);\ \ y(0)=y'(0)=0;\ \text{ where }\ f(t)=\begin{cases}\cos t, & 0\le t\le\pi\\ 0, & t>\pi\end{cases}$$

(a) $y=\frac{1}{2}\left[t\cos t+H\left(t-\pi\right)\left(t-\pi\right)\sin\left(t-\pi\right)\right]$

(b) $y=\frac{1}{2}\left[t\sin t+H\left(t-\pi\right)\left(t-\pi\right)\sin\left(t-\pi\right)\right]$

(c) $y=\frac{1}{2}\left[t\sin t-H\left(t-\pi\right)\left(t-\pi\right)\sin\left(t-\pi\right)\right]$

(d) $y=\frac{1}{2}\left[t\cos t-H\left(t-\pi\right)\left(t-\pi\right)\sin\left(t-\pi\right)\right]$

15. The solution of the integral equation $y(t)=\sin t+2\int_0^t y(u)\cos(t-u)du$ is

(a) $y(t)=te^{-t}$ (b) $y(t)=t^2e^t$

(c) $y(t)=te^t$ (d) $y(t)=t^2e^{-t}$

16. The solution of the integro-differential equation $\frac{dy(t)}{dt}+3y(t)+2\int_0^t y(u)du=t;\ \ y(0)=1$ is

(a) $y(t)=\frac{1}{2}-2e^{-t}+\frac{5}{2}e^{2t}$

(b) $y(t)=\frac{1}{2}-2e^{t}+\frac{5}{2}e^{-2t}$

(c) $y(t)=\frac{1}{2}-2e^{t}+\frac{5}{2}e^{2t}$

(d) $y(t)=\frac{1}{2}-2e^{-t}+\frac{5}{2}e^{-2t}$

17. The solution of the wave equation

$$\frac{\partial^2 y}{\partial t^2}=\frac{\partial^2 y}{\partial x^2};\ \ 0<x<1,\ t>0$$

with the following initial and boundary conditions

$y(x,0+)=\sin\pi x,\ \ 0<x<1,$ $\qquad$ $y_t(x,0+)=0,\ \ 0<x<1$

$y(0,t)=0,\ \ t>0,$ $\qquad$ $y(1,t)=0,\ \ t>0$

is

(a) $y(x,t)=\sin\pi x\cos\pi t$

(b) $y(x,t)=\sin 2\pi x\cos\pi t$

(c) $y(x,t)=\sin\pi x\cos 2\pi t$

(d) $y(x,t)=\sin 2\pi x\cos 2\pi t$

18. The solution of the integral equation $F(t)=t+2\int_0^t \cos(t-u)F(u)du$

(a) $2e^{t}(t-1)-2+t$

(b) $2e^{t}(t+1)+2+t$

(c) $2e^{t}(t-1)+2+t$

(d) $2e^{-t}(t-1)+2+t$

19. The value of the integral $\int_0^\infty e^{-t}\frac{\sin t}{t}dt$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

20. The value of the integral $\int_0^\infty t\,e^{-3t}\cos t\,dt$ is

(a) $\frac{1}{25}$ (b) $\frac{2}{25}$

(c) $\frac{3}{25}$ (d) $\frac{4}{25}$

Answers:

1. b	2. c
3. b	4. a
5. a	6. c
7. d	8. c
9. d	10. a
11. a	12. d
13. a	14. b
15. c	16. d
17. a	18. c
19. b	20. b